



Notes About the Bloch Sphere

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Joint Quantum Institute

and

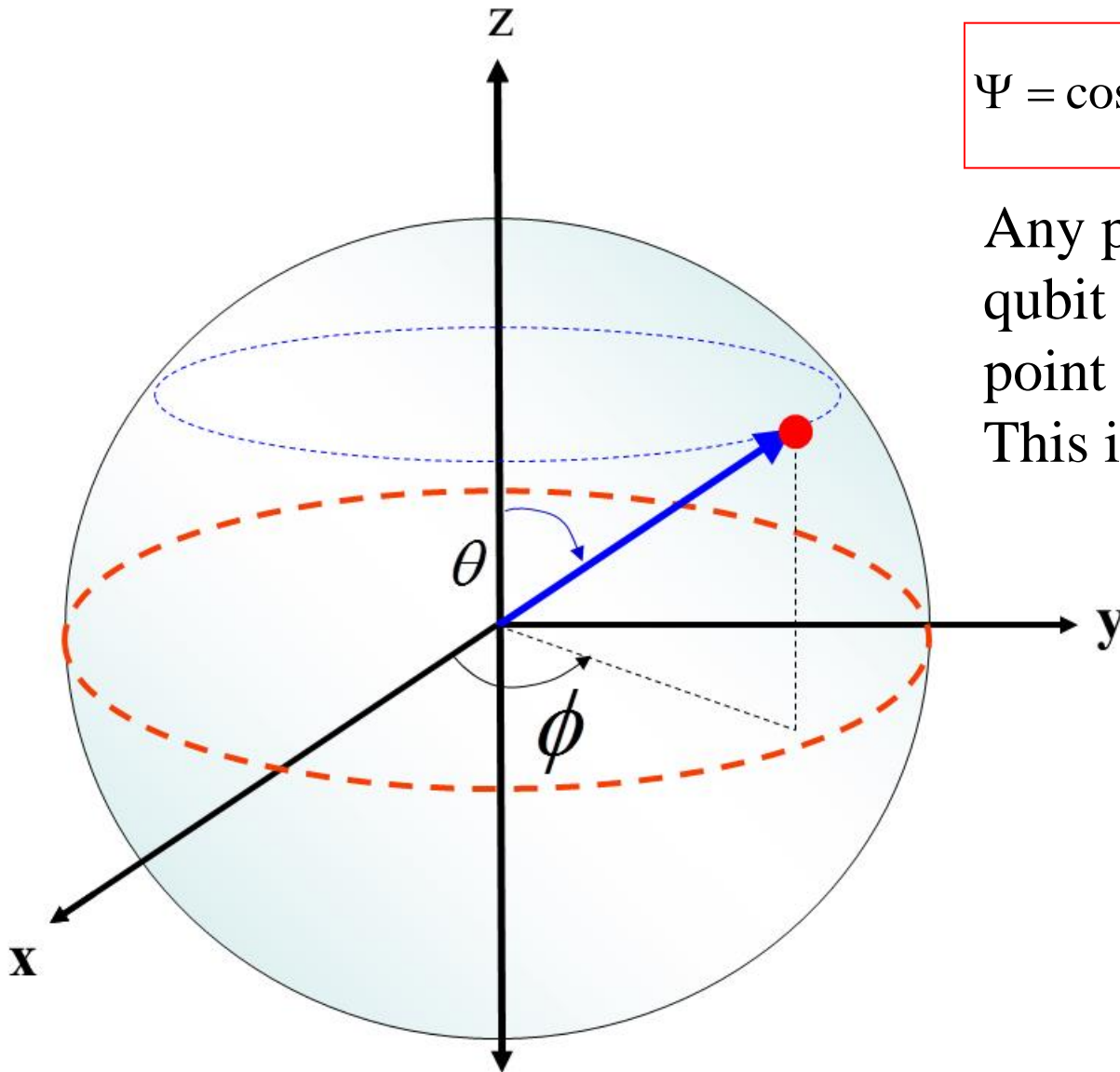
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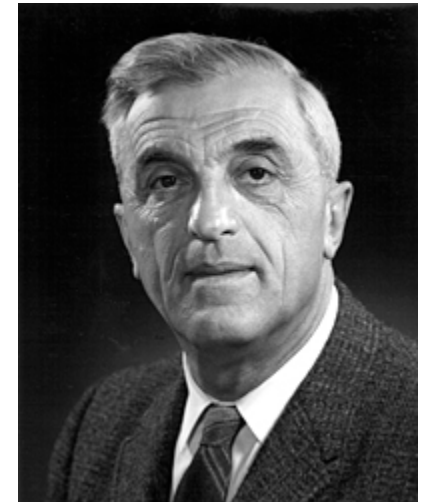


III. Quantum states and gates



$$\Psi = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi} \sin\left(\frac{\theta}{2}\right)|1\rangle$$

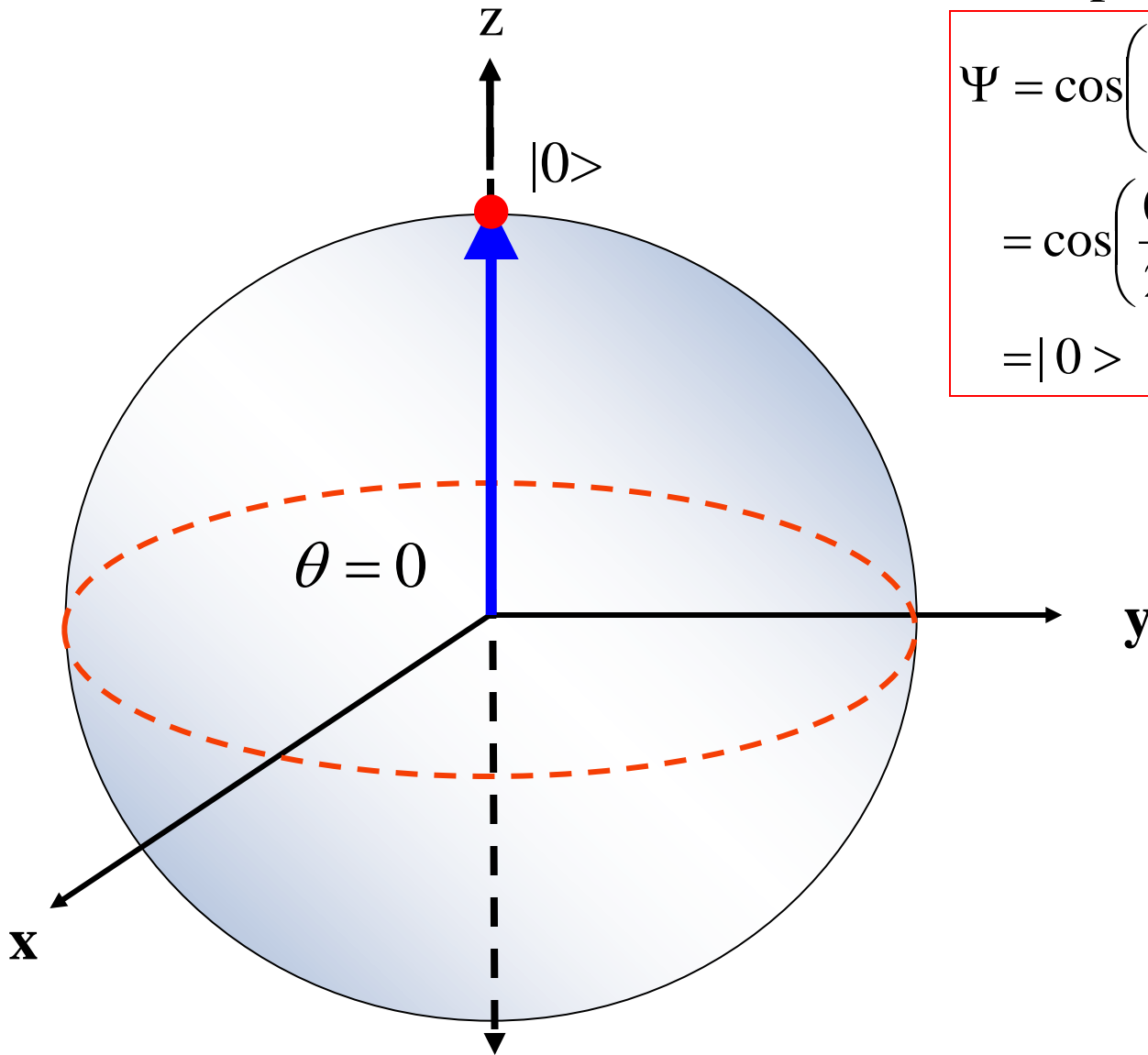
Any pure quantum state of a qubit can be visualized as a point on a unit sphere. This is the “Bloch Sphere”.



(ref. 5.5)

Felix Bloch
Nobel Prize in 1952 for NMR

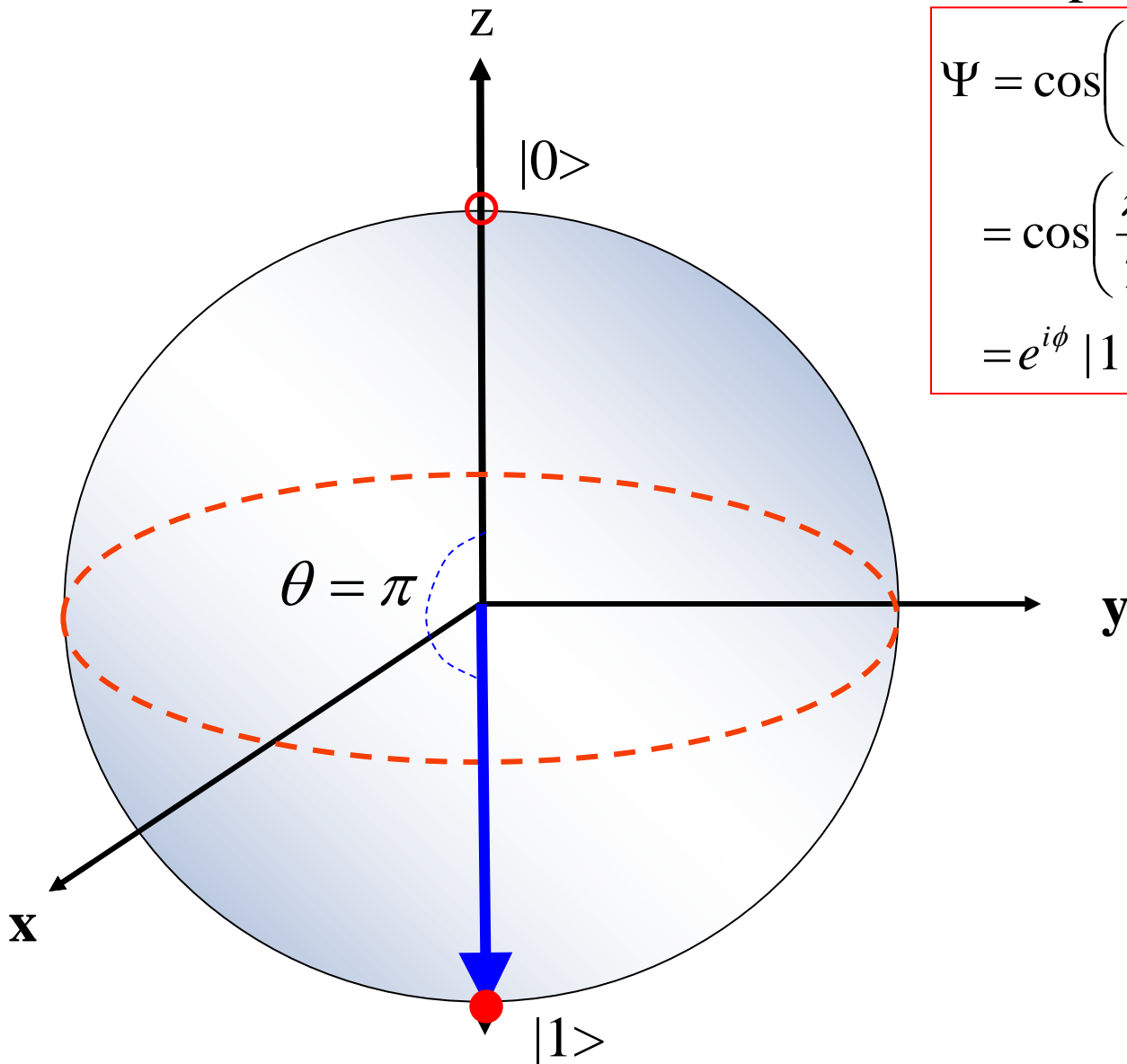
III. Quantum states and gates



Example: $\theta = 0$

$$\begin{aligned}\Psi &= \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle \\ &= \cos\left(\frac{0}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{0}{2}\right)|1\rangle \\ &= |0\rangle\end{aligned}$$

III. Quantum states and gates



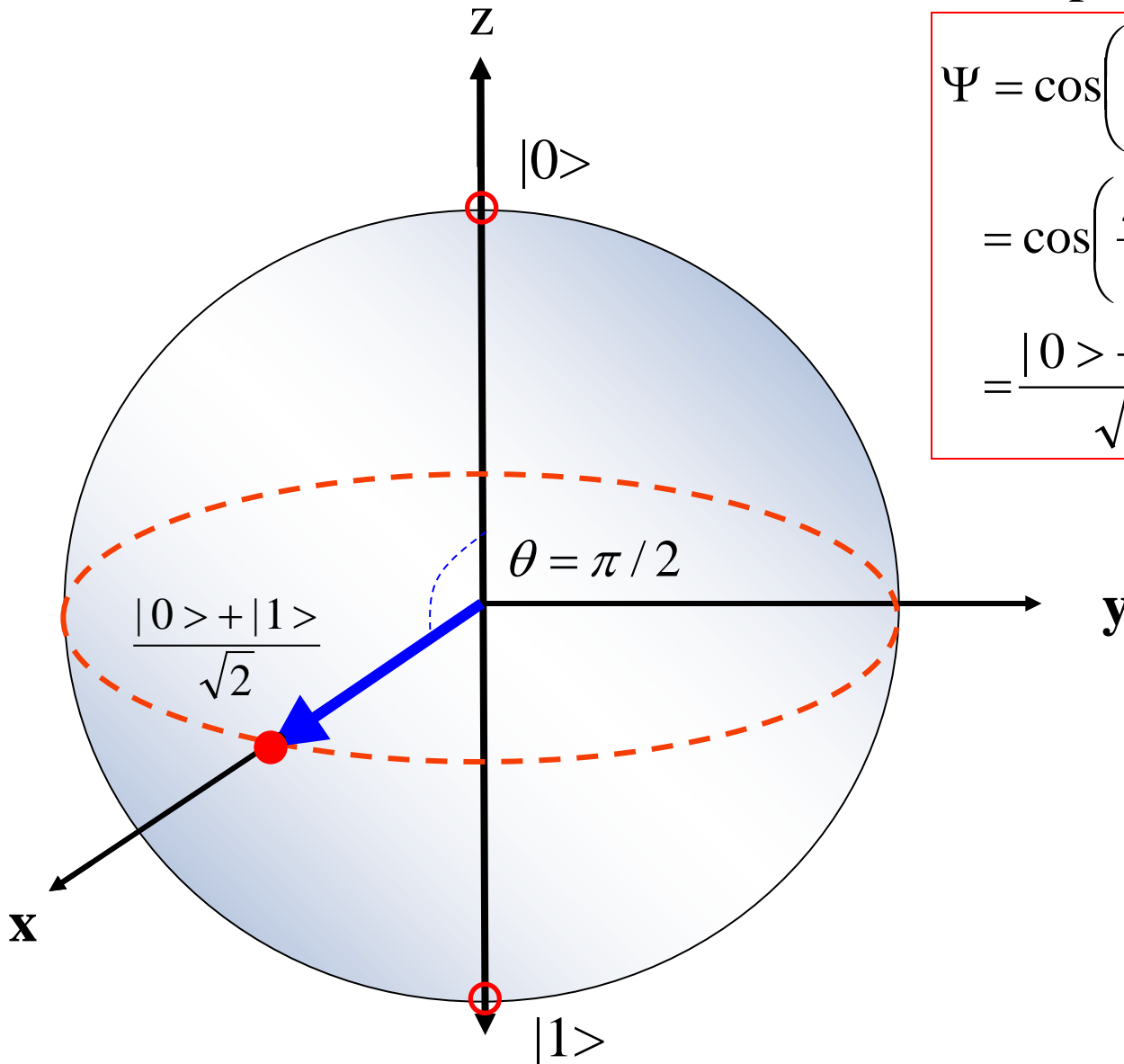
Example: $\theta = \pi$

$$\begin{aligned}\Psi &= \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle \\ &= \cos\left(\frac{\pi}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\pi}{2}\right)|1\rangle \\ &= e^{i\phi}|1\rangle\end{aligned}$$

III. Quantum states and gates

Example: $\theta = \pi/2, \phi = 0$

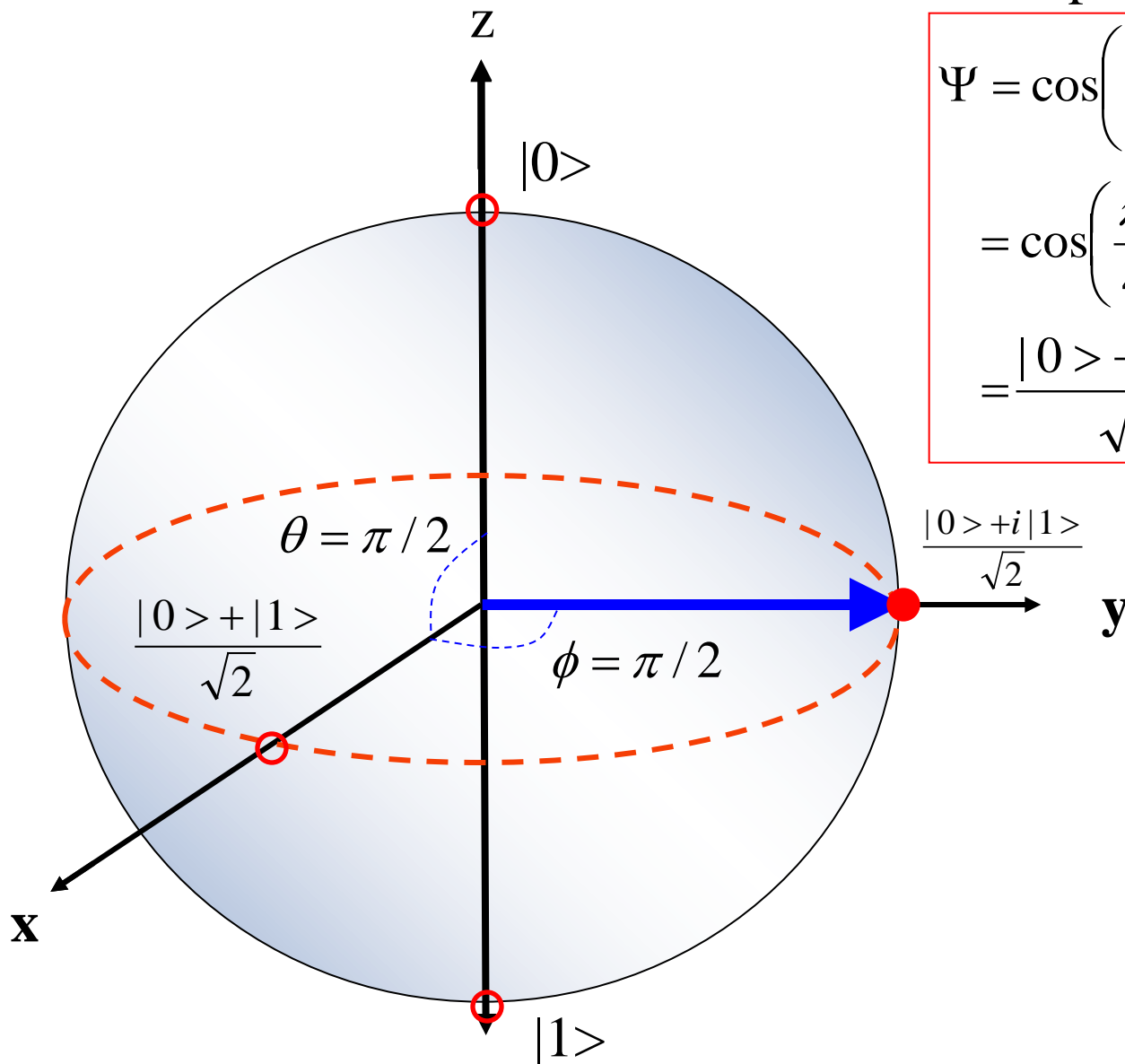
$$\begin{aligned}\Psi &= \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle \\ &= \cos\left(\frac{\pi}{4}\right)|0\rangle + e^{i0}\sin\left(\frac{\pi}{4}\right)|1\rangle \\ &= \frac{|0\rangle + |1\rangle}{\sqrt{2}}\end{aligned}$$



III. Quantum states and gates

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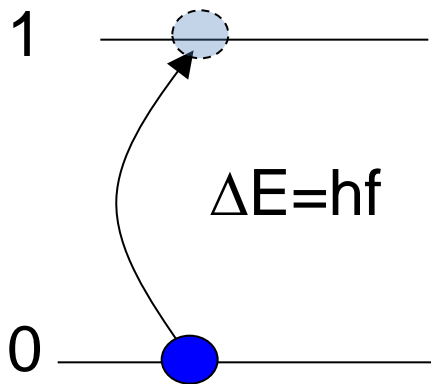
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Implementing Quantum Gates

Rotating states on the Bloch sphere using a “Rabi Oscillation”

- Consider qubit with energy splitting ΔE .
- Apply power (a perturbation) at frequency $f = \Delta E/h$.

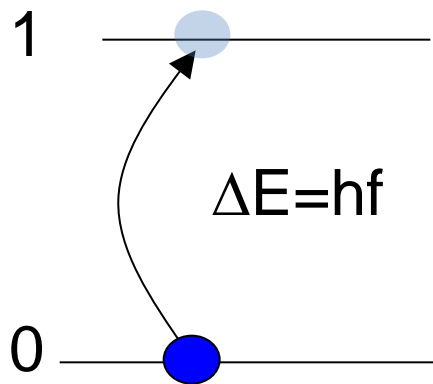


Start in $|0\rangle$ and apply power for short time
--> develop small amplitude to be in $|1\rangle$

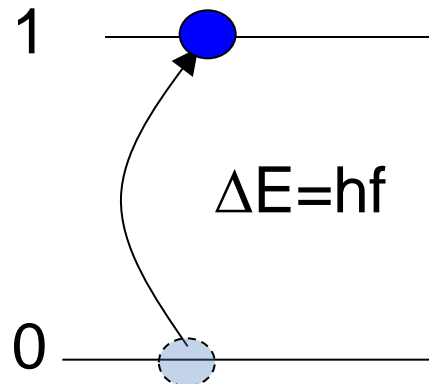
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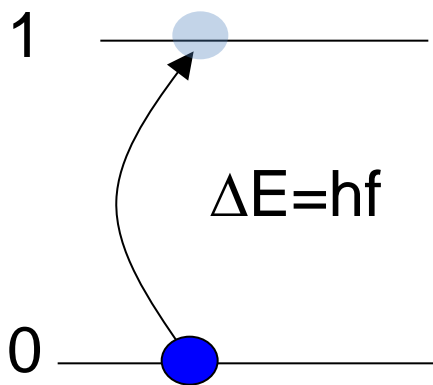


Keep applying power
--> eventually system pumped entirely into 1
(NOT gate or π -pulse)

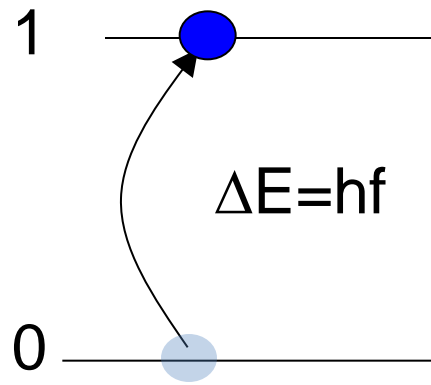
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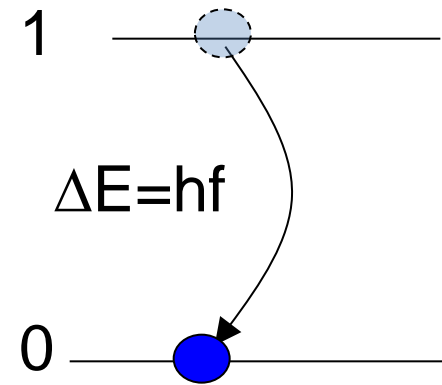
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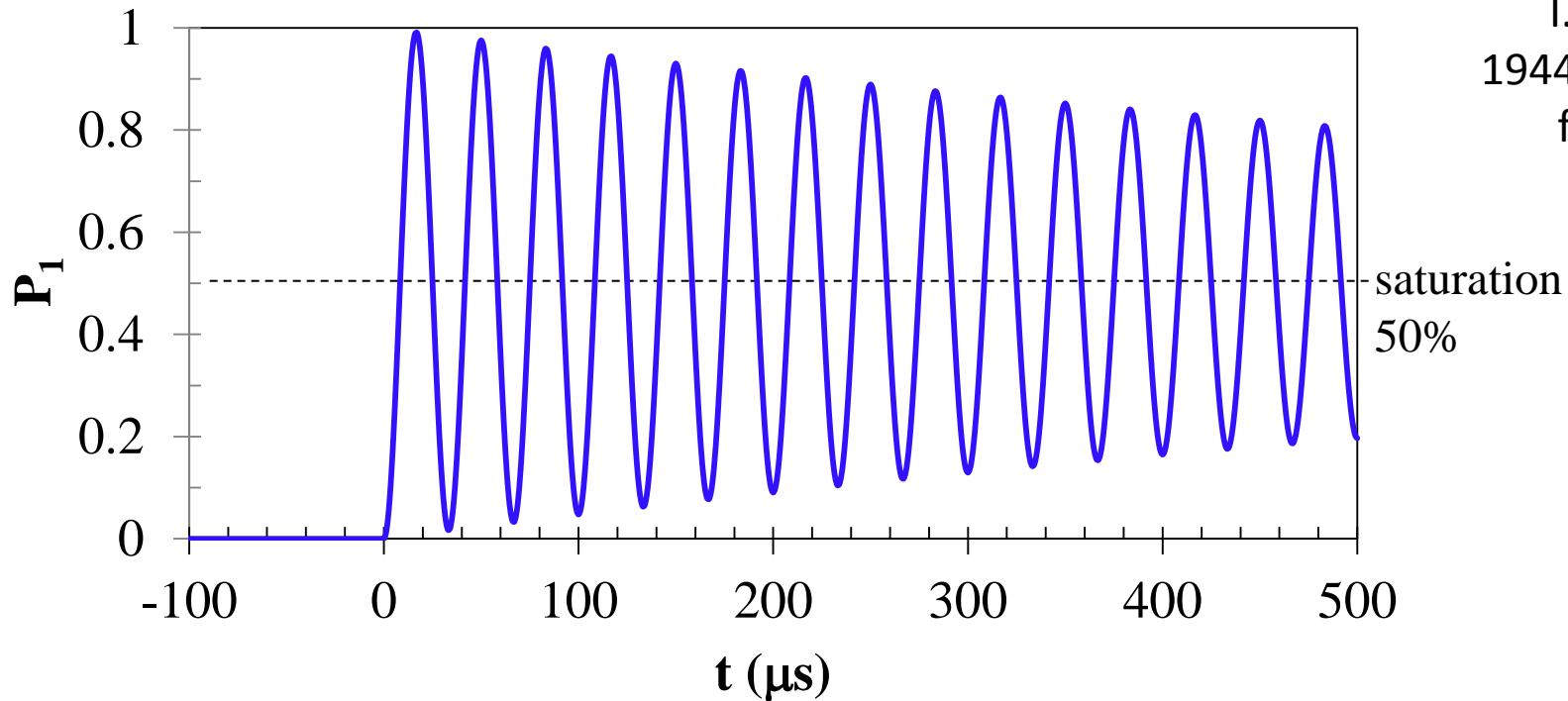
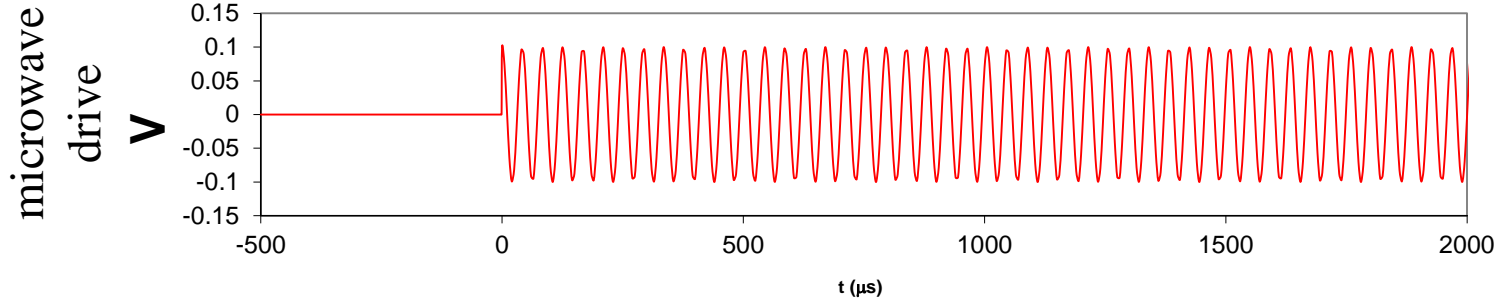


Keep applying power
--> system pumped back down to 0
(stimulated emission)

Rabi oscillations with zero detuning



I. I. Rabi -
1944 Nobel Prize
for NMR
(ref 5.6)



State of system cycles back and forth between $|0\rangle$ and $|1\rangle$ at well-defined rate (Rabi frequency) set by drive power. Stop at appropriate time to make a NOT or \sqrt{NOT}

Spin Echo

